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Evaluation of the Impact of Fertility-Control from Birth-Order Statistics

Introduction

THE study of order of births is necessary to understand the behaviour of the couples towards procreation such as their motivation either to limit their family size or to continue to give births to children without any control. The birth order statistics may then be used to evaluate the impact of fertility-control in a population undergoing a change in character from non-contraceptive to contraceptive.

In an unpublished thesis on 'A Generalised Model of Human Fertility and Mortality, Prasad has suggested five models, one for non-contraceptive population (Type *B),* one for contraceptive population (Type *A)* and three for those populations which are in transition stages, namely transition stage I (Type C_3) transition stage II (Type C_1) and transition stage III (Type C_2). The model which fits the best of all determines the contraceptive or non-contraceptive character of the transition stage of the population. In case of a population transiting from non-contraceptive to contraceptive character, it is assumed that the five models suggested by Prasad would fit the birth order statistics at different times showing the various stages of transition in the order type *B,* type C_3, type C_1, type C_2 and type *A.* The assumption is based on the rationale of these models and is supported by the fact that the population of Japan which has undergone a change in character from non-contraceptive to contraceptive (1949-1967) has passed through all these five stages in the same order.

2. Models

The probabilities of the different birth orders (x) are give below :

x	*Probability*
1	θ_1
2	$\bar{\theta}_1 \theta_2$
3	$\bar{\theta}_1 \bar{\theta}_2 \theta_3$
4	$\bar{\theta}_1 \bar{\theta}_2 \bar{\theta}_3 \theta_4$
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$m-1$	$\bar{\theta}_1 \bar{\theta}_2 \ldots \bar{\theta}_{m-2}\theta_{m-1}$
$m+$	$\bar{\theta}_1 \bar{\theta}_2 \ldots \bar{\theta}_{m-1}\bar{\theta}_m$

where $\theta_1 = p$, the probability of a first order birth $\bar{\theta}_i = 1 - \theta_i$.

The values of $\theta_2, \theta_3, \theta_4, \theta_5, \theta_6 \ldots \theta_i \ldots$ in all the five models are as follows :

	Type B	*Type C_3*	*Type C_1*	*Type C_2*	*Type A*
θ_2 :	$\theta_1+\alpha$	$\theta_1+\alpha$	$\theta_1+\alpha$	$\theta_1+\alpha$	$\theta_1+\alpha$
θ_3 :	$\theta_1+k\alpha$	$\theta_1+k\alpha$	$\theta_1+k\alpha$	$\theta_1+k\alpha$	$\theta_1+k\alpha$
θ_4 :	$\theta_1+(k+c)\alpha$	$\theta_1+(k+c)\alpha$	$\theta_1+k\alpha$	$\theta_1+(k-c)\alpha$	$\theta_1+(k-c)\alpha$
θ_5 :	$\theta_1+(k+2c)\alpha$	$\theta_1+(k+c)\alpha$	$\theta_1+k\alpha$	$\theta_1+(k-c)\alpha$	$\theta_1+(k-2c)\alpha$
θ_6 :	$\theta_1+(k+3c)\alpha$	$\theta_1+(k+c)\alpha$	$\theta_1+k\alpha$	$\theta_1+(k-c)\alpha$	$\theta_1+(k-3c)\alpha$
.					
.					
.					
θ_i	$\theta_1+\{k+(i-3)c\}\alpha$	$\theta_1+(k+c)\alpha$	$\theta_1+k\alpha$	$\theta_1+(k-c)\alpha$,	$\theta_1+\{k-(i-3)c\}\alpha$
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All the models except type C_1 involve four parameters namely $p(=\theta_1)$ α, k and c. Type C_1 involves only three parameters p, α and k. The estimates of these parameters are obtained by maximum likelihood method of estimation. The likelihood equations (given in appendix) are solved by the method of iteration using the trial values of the parameters p, α, k and c, obtained by equating the theoretical and the observed proportions for the first four orders of birth.

In the construction of these models it is assumed that all the couples have same intensity of desire to have the first child whether the population is contraceptive or non-contraceptive or transitional and hence the probability for the first order birth is assumed to be the same in case of all types of population. It is also assumed even in the case of a contraceptive population the accepted slogan is to have three children and hence it is assumed that the nature of the population, contraceptive, non-contraceptive or transitional can be determined only by the differences in probabilities of fourth and higher order births. In the case of purely contraceptive population, the probabilities for fourth and higher order births decline appreciably whereas in other cases it either remains constant (Type C_3 C_1 and C_2) or increases (Type *B)*. In form type C_3 and type C_2 models differ only in the sign of the parameter *C* and so do the models type *B* and type *A*.

Application of the Model

In this section the main objective of the paper, to evaluate the change in behaviour of couples towards reproduction over the period 1955-74, is discussed. The data on the birth orders in the year 1955 are taken from Patna in 1955—A Demographic Sample Study' (1), a report based on the demographic sample survey of Patna conducted by the Department of Statistics, Patna University at the instance of Ministry of Health, Government of India. The same data for 1974 are taken from the complete survey of the *Kadam Kuan* area of Patna which may be considered as the cross-section of the whole of Patna conducted in the first quarter of 1975 by the Post Graduate students of Statistics, Patna University.

As the total number of births in these two years are different, in order to make them comparable the distributions of births are so prorated at various birth orders such that the total is 1000 and it is assumed that the transformation leaves the characteristics of the distributions unaltered. Besides this, the observations at $x = 8$ and beyond are censored.

Births in 1955

Taking into consideration the socio-economic conditions prevailing in and about 1955, including the level of mass consciousness towards limiting size etc. it was thought proper to fit type *B* model (model for non-contraceptive population) and type C_3 model (model for transition stage 1) to the birth order statistics of 1955.

The ML estimates $\hat{p}$, $\hat{\alpha}$, $\hat{k}$, and $\hat{c}$ of the parameters p, α, k and c respectively for the two models are obtained as

Type B	*Type C_3*
$\hat{p} = 0.22211$	$\hat{p} = 0.22214$
$\hat{\alpha} = -0.00349$	$\hat{\alpha} = -0.00348$
$\hat{k} = -14.22197$	$\hat{k} = -15.56326$
$\hat{c} = -9.41071$	$\hat{c} = -16.86535$

The following table shows the observed and expected frequencies of different birth orders for each of the two models.

TABLE 1—GOODNESS OF FIT OF TYPE *B* AND TYPE C_3 MODELS TO THE OBSERVED DISTRIBUTION OF BIRTHS IN 1955 ACCORDING TO ORDER

Birth Order	*Observed*	*Expected frequency*	
(X)	*frequency*	*Type B Model*	*Type C_3 Model*
1	222	222	222
2	170	170	170
3	168	165	168
4	126	134	148
5	112	104	98
6	75	76	65
7	50	52	43
8+	77	77	86
		$X^2 = 1.23761$ $X^2_{3,}$ (-05) = 7.815	$X^2 = 8.89012$

As the value of x^2 for type *B* is insignificant and that of type C_3 is significant, it is inferred that type *B* model fits the birth order statistics in 1955. It means that the population was non-contraceptive in character in 1955.

Births in 1974

Considering the continuous and vigorous efforts made during 1955-74 by the Government of India through the national programme of family planning to limit the family size it may be assumed that the character of population has shifted from non-contraceptive to a transitional stage in 1974. Keeping these in view it was decided to fit the models of the transition stages to the distribution of birth order statistics of 1974. But when the model type C_2 was applied the estimate of the parameter *c* came negative which shows that the population as yet not reached transition stage III. The estimates of the parameters of the three models of transition stages are obtained as

		Type C_3	*Type* C_1
$\hat{p}$	:	0.34202	0.34208
$\hat{\alpha}$	:	0.01970	0.01970
$\hat{k}$	:	0.64781	0.95257
$\hat{c}$	:	0.50964	—

The observed frequencies and expected frequencies of different birth-orders for all the three transition stage models are shown in the following table.

TABLE 2—GOODNESS OF FIT OF THE TRANSITION STAGE MODELS TO THE OBSERVED DISTRIBUTION OF BIRTHS IN 1974 ACCORDING TO ORDER

Birth Order	*Observed frequency*	*Expected frequency*	
		Stage I (*Type* C_3)	*Stage II* (*Type* C_1)
1	342	342	342
2	238	238	238
3	149	149	151
4	95	99	97
5	66	63	64
6	42	40	40
7	25	25	25
8+	43	44	43
		$X^2 = .42721$ d.f. = 3	$X^2 = .23023$ d.f. = 4

The value of x^2 in case of type C_1 is found less than that in case of type C_3 which determines the character of the population in transition stage II. The model for contraceptive population is not tried as the population has not reached as yet even the transition stage III. Thus it may be inferred that the population has not yet crossed the transition stage II.

The fact that there has been an appreciable shift in the attitude and practice of people in favour of family planning is also reflected in the mean birth orders for the years 1955 and 1974. The mean birth order in 1955 is 3.546 and that in the year 1974 is 2.746. The variance of the birth order in 1955 is 4.7348 and that in the year 1974 is 3.7535. The decrease m variance of birth order during this period shows greater concentration of birth orders round the mean.

The present study is based on the birth order statistics of Patna in the years 1955 and 1974. But it is felt that the inferences drawn should also be true for the whole of the country. Although Patna is one of the big cities of the country, it is yet influenced by, as it also influences, the rural characteristics. Most of the people are recent migrants from the villages who have not yet been able to shake off the rural attitudes, habits, and mode of living. Being the capital and an important business centre of the state this city is also cosmopolitan in character. There are people belonging to the different levels of socio-economic conditions and thus it would not be wrong to assume that the population of the country as a whole has achieved the transition stage II.

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References

1. Lal, D. N., 1955, *Patna in 1955—A Demographic Sample Study,* Demographic Research Centre, Patna University.
2. Prasad, R. N., 1972, A generalised model of human fertility and mortality, *Ph.D. Thesis* (Unpublished), Patna University.

APPENDIX

The likelihood functions for the type A and type B models are given by

$$L = p^{f_1} q^{N-f_1} (p+\alpha)^{f_2} (q-\alpha)^{N-f_1-f_2} \times$$

$$\times \prod_{i=3}^{7} (p + \overline{k \pm \overline{i-3}c\alpha})^{f_i} (q - \overline{k \pm \overline{i-3}c\alpha})^{N - \sum_{j=1}^{i} f_j}$$

where $f_1, f_2, \ldots$ are the frequencies $f_1 + f_2 + f_3 + \ldots + f_{8+} = N$ and $q = 1 - p$.

Similarly the likelihood functions for type C_2 and type C_3 are given by

$$L = p^{f_1} q^{N-f_1} (p+\alpha)^{f_2} (q-\alpha)^{N-f_1-f_2} (p+k\alpha)^{f_3} (q-k\alpha)^{N-f_1-f_2-f_3}$$

$$\times (p + \overline{k \pm c\alpha})^{\sum_4^7 f_j} (q - \overline{k \pm c\alpha})^{f_5+2f_6+3f^7+4f_8+}$$

For the type C_1 the likelihood function is given by

$$L = p^{f_1} q^{N-f_1} (p+\alpha)^{f_2} (q-\alpha)^{N-f_1-f_2} (p+k\alpha)^{\sum_3^7 f_j} (q-k\alpha)^{f_4+2f_5+3f_6+4f_7 5f_8+}$$

The ML estimates $\hat{p}\hat{\alpha}\hat{k}$ and $\hat{c}$ of the parameters p, α, k and c (p, α and k in type C_1) were obtained by the method of iteration from the following equations. $p_0\alpha_0k_0$ and c_0 are the trial values obtained by equating the observed and theoretical proportions for the first four orders of birth

$$\left.\frac{\partial \log L}{\partial p}\right|_{\hat{p}\hat{\alpha}\hat{k}\hat{c}} = 0 = \left.\frac{\partial \log L}{\partial p}\right|_{p_0\alpha_0k_0c_0} + (\hat{p} - p_0)\left.\frac{\partial^2 \log L}{\partial p^2}\right|_{p_0\alpha_0k_0c_0}$$

$$+ (\hat{\alpha} - \alpha_0)\left.\frac{\partial^2 \log L}{\partial\ \partial\alpha}\right|_{p_0\alpha_0k_0c_0} + (\hat{k} - k_0)\left.\frac{\partial^2 \log L}{\partial p \partial k}\right|_{p_0\alpha_0k_0c_0}$$

$$+ (\hat{c} - c_0)\left.\frac{\partial^2 \log L}{\partial p \partial c}\right|_{p_0\alpha_0k_0c_0}$$

$$\left.\frac{\partial \log L}{\partial \alpha}\right|_{\hat{p}\hat{\alpha}\hat{k}\hat{c}} = 0 = \left.\frac{\partial \log L}{\partial \alpha}\right|_{p_0\alpha_0k_0c_0} + (\hat{p} - p_0) \left.\frac{\partial^2 \log L}{\partial p\partial \alpha}\right|_{p_0\alpha_0k_0c_0}$$

$$+ (\hat{\alpha} - \alpha_0) \left.\frac{\partial^2 \log L}{\partial \alpha^2}\right|_{p_0\alpha_0k_0c_0} + (\hat{k} - k_0) \left.\frac{\partial^2 \log L}{\partial \alpha\partial k}\right|_{p_0\alpha_0k_0c_0}$$

$$+ (\hat{c} - c_0) \left.\frac{\partial^2 \log L}{\partial \alpha\partial c}\right|_{p_0\alpha_0k_0c_0}$$

$$\left.\frac{\partial \log L}{\partial k}\right|_{\hat{p}\hat{\alpha}\hat{k}\hat{c}} = 0 = \left.\frac{\partial \log L}{\partial k}\right|_{p_0\alpha_0k_0c_0} + (\hat{p} - p_0) \left.\frac{\partial^2 \log L}{\partial p\partial k}\right|_{p_0\alpha_0k_0c_0}$$

$$+ (\hat{\alpha} - \alpha_0) \left.\frac{\partial^2 \log L}{\partial \alpha\partial k}\right|_{p_0\alpha_0k_0c_0} + (\hat{k} - k_0) \left.\frac{\partial^2 \log L}{\partial k^2}\right|_{p_0\alpha_0k_0c_0}$$

$$+ (\hat{c} - c_0) \left.\frac{\partial^2 \log L}{\partial k\partial c}\right|_{p_0\alpha_0k_0c_0}$$

$$\left.\frac{\partial \log L}{\partial c}\right|_{\hat{p}\hat{\alpha}\hat{k}\hat{c}} = 0 = \left.\frac{\partial \log L}{\partial c}\right|_{p_0\alpha_0k_0c_0} + (\hat{p} - p_0) \left.\frac{\partial^2 \log L}{\partial k\partial c}\right|_{p_0\alpha_0k_0c_0}$$

$$+ (\hat{\alpha} - \alpha_0) \left.\frac{\partial^3 \log L}{\partial \alpha\partial c}\right|_{p_0\alpha_0k_0c_0} + (\hat{k} - k_0) \left.\frac{\partial^2 \log L}{\partial k\partial c}\right|_{p_0\alpha_0k_0c_0}$$

$$+ (\hat{c} - c_0) \left.\frac{\partial \log L}{\partial c^2}\right|_{p_0\alpha_0k_0c_0}.$$